FREQUENCY STABILITY IN PRECISION OSCILLATORS

Extremely stable sources of high-frequency and microwave signals are required for many applications — radar systems and spectroscopy, for instance. Such signals are often derived from a standard-frequency quartz-crystal oscillator. When the frequency of a signal is multiplied, its signal-to-noise ratio is decreased proportionately. Hence, extreme purity of frequency spectrum is demanded of the primary oscillator.

A PERFECT OSCILLATOR would produce one invariant frequency and therefore would have a spectrum consisting of a single line of infinitesimal width (Fig. 1). Any real oscillator produces an output frequency that deviates to some extent from an average value. In the case of a high-precision unit, this deviation is characterized by a modulation index $m << 1$. Therefore, narrow-band FM theory applies [1], and modulation at a certain rate produces a single pair of sideband components (Fig. 2). The ratio of the amplitude of these components to that of the carrier is

$$\begin{aligned} N/S &= E_N/E_S \\ &= m/2 = \Delta f/2f_m \end{aligned} \tag{1}$$

where N/S is the noise-to-signal ratio, E_N is the sideband (noise) amplitude, E_S is the carrier (signal) amplitude, Δf is the maximum frequency deviation, and f_m is the modulating frequency. The value of Δf is determined by the sensititivy of the oscillator frequency to the source of modulation. Frequency multiplication has the effect of multiplying the modulation index (and sideband amplitude) by the multiplication factor while the frequency of modulation is unchanged. [2] Thus, the signal-to-noise ratio is degraded by the multiplication factor.

Sideband components are often caused by modulation at the power-line frequency and its harmonics. These components can be reduced in the design of the oscillator by proper shielding and the avoidance of ground loops.

In the case of quartz-crystal oscillators, mechanical vibrations can also cause discrete sideband components in the output. If the acceleration constant of the crystal is known, the amplitudes of these sidebands can be calculated from Eq (1). [3]

Frequency modulation due to random noise is a fundamental limitation of the spectral purity of a frequency source. The spectrum of a noise-modulated oscillator has a continuous distribution of sideband components, as shown in Fig. 3.

The presence of sideband components in the spectrum of an oscillator indicates that the instantaneous frequency of the oscillator deviates from its average value at a rate corresponding to the location of the sideband component. Conversely, the observation of a frequency deviation indicates the presence of sideband components. Thus it is possible to specify an oscillator's performance with regard to either its spectrum or its frequency variation. Both characteristics contain the same information presented in different ways. However, since the transformation from one to the other is difficult, both should be specified.

Spectrum and short-term frequency-stability data are usually obtained by means of measurements made on an audio-frequency beat note, established between two oscillators after multiplication into the microwave region to provide better resolution. Great care must be taken in the design of the multiplier chain to ensure that discrete and random noise introduced by the system itself does not equal or exceed that of the oscillators. In addition, the multiplier chain must have sufficient bandwidth to permit measurement of the frequency-deviation rates being considered.

A wave analysis of the beat note provides the spectrum of the two oscillators, the noise-to-signal ratio as a function of frequency from the carrier (Fig. 3). If the two oscillators are identical, an additional 3 db is subtracted for one unit.

If the noise-to-signal ratio at one carrier frequency is known, it can be approximated at another frequency by the following equation (as long as the noise is more than 20 db down):

$$(N/S)_2 = 20 \log_{10}(f_2/f_1) + (N/S)_1 \tag{2}$$

where $(N/S)_1$ is the noise-to-signal ratio in db at f_1, and $(N/S)_2$ is the ratio in db at f_2. Equation (2) shows that the ratio is degraded by 20 db for a frequency multiplication of ten.

The noise-to-signal ratio can be normalized to a 1-cps bandwidth. Thus

$$(N/S)_{norm} = (N/S)_B - 10\log_{10} B \qquad (3)$$

where $(N/S)_B$ is the ratio in bandwidth B.

The noise distribution at each side of a carrier is a result of the superposition of the effects of several noise sources in the oscillator. The noise sources which should be considered are: the thermal noise of the crystal, filtered by the crystal network; the wideband noise at the input of the amplifier chain, filtered by the amplifier tuned circuits; and the $1/f$ noise of the semiconductors.

Noise inside the oscillator loop causes a permanent perturbation of oscillator phase, the nature of which is a "random walk." Additive noise, however, averages to zero.

It has been shown [4] that the equivalent noise resistance of a quartz crystal is the same as its effective series resistance at the operating temperature. Therefore, the crystal noise voltage E_{NX} is

$$E_{NX} = \sqrt{4kTBR} \qquad (4)$$

where k is Boltzmann's constant, T is absolute temperature, B is bandwidth in which E_{NX} is measured, and R is effective crystal series resistance. The carrier, or oscillator signal voltage, is

$$E_S = \sqrt{PR} \qquad (5)$$

where P is crystal driving power. The noise-to-signal ratio of the quartz-crystal oscillator itself is

$$\begin{aligned}(N/S)_X &= E_{NX}/E_S \\ &= \sqrt{4kTBR}/\sqrt{PR} = \sqrt{4kTB}/\sqrt{P} \qquad (6)\end{aligned}$$

It is not possible to improve the ratio simply by increasing the driving power. The oscillator frequency is affected by changes in the crystal driving current, and this level sensitivity increases as the square of the crystal current. [3] Since the thermal noise is inversely proportional to the drive current, there is an optimum operating point dictated by the performance of the level-control circuit.

The crystal resonator can be considered to be a filter for the wideband noise voltage E_{NX}. The crystal's thermal noise contribution in the oscillator spectrum therefore has a narrow-band characteristic (Fig. 4) of

$$E_{NX}' = E_{NX}/\sqrt{1 + (2Q\,\Delta f/f_o)^2} \qquad (7)$$

where E_{NX}' is crystal noise sideband amplitude, Q is crystal quality factor, f_o is carrier frequency, and Δf is frequency deviation from f_o. For example, a 5-Mc crystal with $Q = 2.5 \times 10^6$ acts as a filter with 2-cps 3-db bandwidth.

The spectrum of Fig. 3 shows the first visible sidebands at -45 db in a 10-cps bandwidth at 10 Gc, 100 cps away from the carrier. This corresponds to a value of -124 db in a 1-cps bandwidth at 5 Mc for one oscillator. For this oscillator,

$$E_{NX}/E_S = \sqrt{4kT/P}$$

If $T = 350°K$ and $P = 0.7 \times 10^{-6}$, $E_{NX}/E_S = -136$ db in a 1-cps bandwidth at 5 Mc.

The measured noise is thus 12 db above that predicted by the crystal noise model, and can be ac-

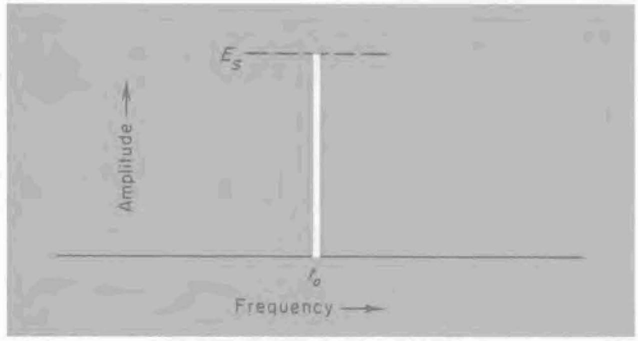

Fig 1 – Spectrum of a perfect oscillator.

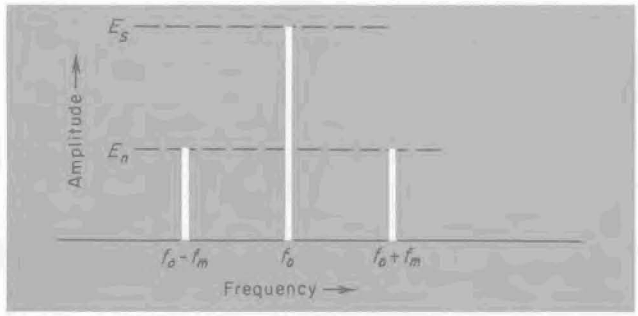

Fig 2 – Spectrum of oscillator with single pair of discrete sidebands.

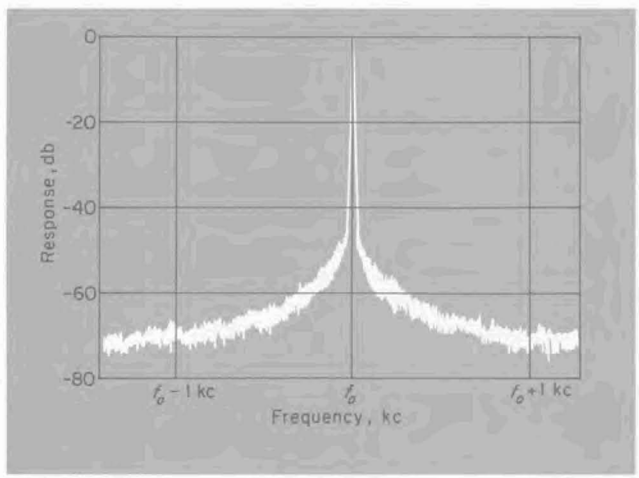

Fig 3 – Spectrum of two 5-Mc quartz-crystal oscillators measured at 10 Gc with 10-cps analyzer bandwidth.

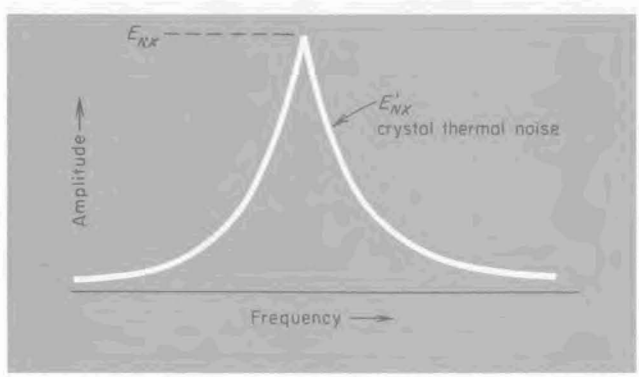

Fig 4 – Contribution of crystal thermal noise to oscillator spectrum.

counted for as the contribution from other noise sources of the oscillator circuit.

Near the carrier, the oscillator spectrum is affected by the random walk of carrier phase due to the perturbing effects of thermal and shot noise [5]; also, it is affected by $1/f$ noise of the oscillator semiconductors. It is the influence of the $1/f$ noise that is most important. The high-frequency transistors required in the oscillator circuit easily account for 12 db of noise 100 cps from the carrier.

Farther away from the carrier, additive wideband noise is the dominant influence on oscillator spectral purity. Oscillator and amplifier electronic circuits contribute wideband white noise, which determines the ultimate level of noise sidebands far from the carrier. The selectivity of the amplifier chain determines the frequency characteristic of this noise. A passive-output crystal filter is often added to an oscillator to reduce its wideband noise level and thus improve its spectral purity. Such a filter is required to obtain good short-term stability over very short averaging times since this performance is largely determined by noise components far from the carrier. The spectrum near the carrier is not affected, however, since the main spectral line of a precision oscillator is much narrower than that of any passive filter that could be employed.

The actual oscillator spectrum shown in Fig. 3 is then obtained by superimposing the wideband noise characteristic on the carrier spectral line of Fig. 4.

Short-Term Stability. Short-term stability measurements of oscillator output frequency are made by using an electronic counter to measure frequency or period on the beat note between two oscillators. The rms fractional frequency deviation is determined as a function of averaging time (the averaging time is equal to the interval during which the counter gate is open.) If the oscillators are identical, the performance of a single unit is obtained by dividing the measured values by $\sqrt{2}$.

An expression for the short-term rms frequency deviation of an oscillator contains terms that describe the effect of each noise source and its variation with averaging time. Complete expressions which take into account the effect of each of the noise sources previously discussed have been derived. [6, 7] The predominant terms are those associated with additive white noise and $1/f$ noise, and these lead to an expression for the rms (fractional) frequency deviation:

$$\sigma = \frac{1}{\omega_o \tau}\left[\frac{P_N}{P_S}(1 - e^{-\omega_1 \tau})\right]^{1/2} + C \tag{8}$$

where P_N is a wideband noise power, P_S is signal power, ω_o is carrier frequency in radians per sec, τ is averaging time, ω_1 is half of the bandwidth of the output filter, and C is the $1/f$ noise factor. Also

$$P_N = 4kTR_N B_0 = 8kTR_N \omega_1 \tag{9}$$

where R_N is the equivalent noise resistance at the amplifier input and B_0 is the 3-db bandwidth of the output filter, or $2\,\omega_1$.

Figure 5 is a plot of σ versus τ from Eq (8), where the oscillator has the following characteristics:

$P_S = 0.7 \times 10^{-6}$ watt
$\omega_o = 2\pi(5 \times 10^6)$ rad/sec $= 31.4 \times 10^6$ rad/sec
$\omega_1 = B_0/2 = 2\pi \times 100/2 = 3.14 \times 10^3$ rad/sec
$R_N = 10$ ohms
$kT = 5 \times 10^{-21}$ watt-sec

Equation (8) and Fig. 5 show that σ varies as $1/\tau$ for $\omega_1\tau \gg 1$ and as $1/\sqrt{\tau}$ for $\omega_1\tau \ll 1$.

There is no theory available to predict the magnitude of the factor C which is the contribution of $1/f$ noise, and the curve in Fig. 5 is drawn to pass through the measured point for $\tau = 1$ sec. Oscillator short-term performance is thus limited by the noise of the active circuit elements and by the characteristic of the output filter.

The rms phase deviation $\Delta\phi$ at a certain frequency can be found from a plot of $\Delta f/f$ versus τ from the relationship

$$\Delta\phi = 2\pi f \frac{\Delta f}{f}\tau \tag{10}$$

Short-term stability specifications must be expressed in precise statistical terms to be meaningful. Therefore, the rms fractional frequency deviation (standard deviation) must be stated as being less than a certain value with a certain confidence. This information should be given as a function of averaging time.

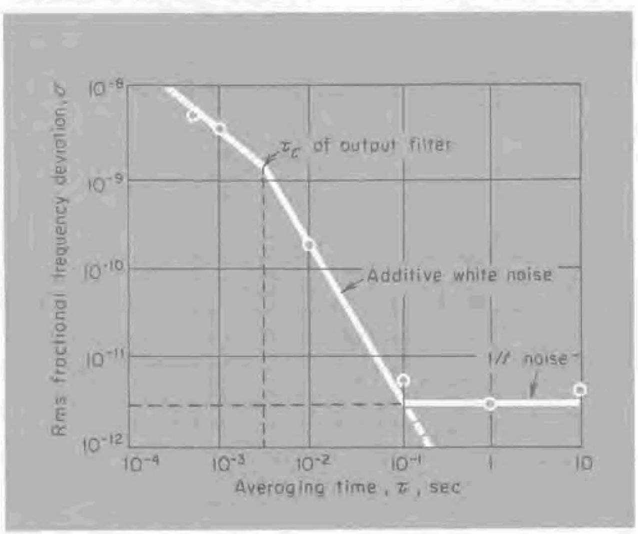

Fig 5 – Root-mean-square fractional frequency deviation versus averaging time for the oscillator whose spectrum is shown in Fig 3. Circled points are measured values of σ.

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